Math 121 **Test #3 Solutions** **Spring 2005**

1. (a) This is a basic probability problem using the normal tables. First, find the z-score: $\frac{56000-50000}{12000} = 0.5$. Then find $P(Z > 0.5)$. That is given by `normalcdf(0.5,E99)` $= 0.3085$.

   (b) This is similar to part (a), but by the Central Limit Theorem, $\overline{x}$ has a distribution that is normal with mean 50000 and standard deviation $\frac{12000}{\sqrt{16}} = 3000$. Then compute

$$P(\overline{X} > 56000) = P(Z > \frac{56000-50000}{3000}) = P(Z > 2) = 0.0228.$$

   There is an alternate method of working this problem that I had not anticipated, but is perfectly valid. Several people used this method. Select `ZTest` on the TI-83 and enter 50000 for μ_0, 12000 for σ, 56000 for $\overline{x}$, and 1 (in part (a)) or 16 (in part (b)) for n. Select $\mu > \mu_0$ as the alternative hypothesis and press Calculate. The p-value is the answer.

2. (a) The hypotheses are

$$H_0 : p = 0.28$$
$$H_1 : p < 0.28$$

   (The null hypothesis is not $p_1 = p_2$ because we are not comparing one sample proportion to another sample proportion. There is only one sample in this problem.)

   (b) First, $\hat{p} = \frac{130}{500} = 0.26$. The value of the test statistic is

$$z = \frac{0.26 - 0.28}{\sqrt{\frac{(0.28)(0.72)}{500}}} = \frac{0.02}{0.0201} = -0.9960.$$

   (c) The p-value is $P(Z < -0.9960) = 0.1596$.

   (d) Since $0.1596 > 0.05$, the results are *not* statistically significant.

   (e) The conclusion is that the smoking rate among men in Great Britain in 2004 is still 28%.

   This problem could be worked using `1-PropZTest` on the TI-83.

3. (a) The sample proportion is $\hat{p} = \frac{55}{100} = 0.55$. Therefore, our estimate of the standard deviation of $\hat{p}$ is $\sqrt{\frac{(0.55)(0.45)}{100}} = 0.04975$. Then the 95% confidence interval is

$$0.55 \pm (1.960)(0.04975) = 0.55 \pm 0.09751.$$

   (b) The margin of error is 0.09751.

   This problem could be worked using `1-PropZInt` on the TI-83. In that case, you get $(0.45249, 0.64751)$ for the confidence interval. The margin of error is $\frac{0.64751-0.45249}{2} = 0.09751$.

4. (a) The hypotheses are

$$H_0 : \mu = 15.90$$
$$H_1 : \mu > 15.90$$

   (b) The population (of hourly earnings) is normal and σ is unknown. Therefore, the t distribution is the appropriate choice. Furthermore, $n < 30$, so the standard normal distribution would not be appropriate.

   (c) The value of the test statistic is

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{16.25 - 15.90}{2.20/4} = \frac{0.35}{0.55} = 0.6364.$$

   (d) The p-value is $P(T > 0.6364)$. If you use the t tables, with 15 degrees of freedom, you get that $0.20 < p\text{-value} < 0.30$. If you use the TI-83, you get `tcdf(0.6364,E99,15)` $= 0.2671$.

   (e) The conclusion is that the average hourly earnings in April 2005 is not greater than \$15.90.

   This problem could be worked using `TTest` on the TI-83.

5. This was meant to be a "gimme." The samples are independent because there is no logical way to associate a man in the first group with any particular man in the second group. Our examples of paired samples in class included such things as a sample of women and a sample of their husbands (obvious pairing: husband with wife) and a sample of patients' conditions before treatment and the sample of *those same patients'* conditions after treatment (obvious pairing: patient with himself, before and after). The terminology "paired sample" does not mean literally that the *samples* are paired. It means that the individuals in the samples are paired off in some logical way.

6. (a) The assumption appears to be justified because the sample standard deviations are very close, 74.6 vs. 72.4.

   (b) Use the formula and calculate

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(19)(74.6^2) + (9)(72.4^2)}{28}} = 73.9.$$

   (c) **Step 1:** The hypotheses are

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 > \mu_2$$

   Step 2: $\alpha = 0.05$.

   Step 3: Use the pooled estimate of σ to calculate t:

$$t = \frac{556 - 483}{73.9\sqrt{\frac{1}{20} + \frac{1}{10}}} = 2.551.$$

Step 4: The p-value is $P(T > 2.551)$ with 28 degrees of freedom. If you use the t table, you get $0.005 < p\text{-value} < 0.01$. If you use the TI-83, you get `tcdf(2.551,E99,28)` $= 0.008255$.

Step 5: The conclusion is that the men's average SAT-M score is higher than the women's.

This problem could be worked using `2-SampTTest` on the TI-83, which also gives the pooled estimate of σ, identified as `Sxp`.

7. The point estimate of the difference is $556 - 483 = 73$. Use the pooled estimate of σ, namely $s_p = 73.9$. Look up the coefficent t^* in the t table, row 28, column 0.05. The value is $t^* = 1.701$. Then the 90% confidence interval is

$$73 \pm (1.701)(73.9)\sqrt{\frac{1}{20} + \frac{1}{10}} = 73 \pm 48.68.$$

This problem could be worked using `2-SampTInt` on the TI-83. The interval obtained is $(24.311, 121.69)$.

8. **Step 1:** The hypotheses are

$$H_0 : p_1 = p_2$$
$$H_1 : p_1 > p_2$$

Step 2: $\alpha = 0.05$.

Step 3: First, find the pooled estimate of p, the common proportion (assuming H_0). There were $0.28 \times 1000 = 280$ men and $0.24 \times 600 = 144$ women who smoked. Therefore,

$$\hat{p} = \frac{280 + 144}{1000 + 600} = \frac{424}{1600} = 0.265.$$

Next, estimate the standard deviation of $\hat{p}$:

$$\sigma_{\hat{p}} = \sqrt{(0.265)(0.735)\left(\frac{1}{1000} + \frac{1}{600}\right)} = 0.02279.$$

Now we may compute the value of the test statistic:

$$z = \frac{0.28 - 0.24}{0.02279} = 1.7551.$$

Step 4: The p-value is $P(Z > 1.7551) = 0.03962$.

Step 5: The conclusion is that the proportion of men who smoked in Great Britain in 2003 is higher than the proportion of women who smoked.

This problem could be worked using `2-PropZTest` on the TI-83.